LOT-SIZING IN MRP WITH CAPACITY CONSIDERATIONS

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to

my parents

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CERTIFICATE

Certified that the work on Lot-Sizing in MRP with Capacity Considerations, by Mr. J. Suderson, has been carried out under our supervision and has not been submitted elsewhere for a degree.

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CONTENTS

ABSTRACT

In the present work, lot sizing algorithms with capacity consideration are studied. The first part of the work considers the formulation of multi-item, multi-facility capacity constrained lot sizing problem. The objective is to minimize the total cost comprised of holding costs and two types of setup costs - major and minor. Two heuristic methods are developed for solving the problem. The first heuristic assumes that items can be shifted to some other period as a lot only. The second heuristic considers the possibility of splitting the lot and shifting the items. The accuracy and efficiency of the heuristics are studied by comparing the solution with that of a reference problem with no capacity considerations. The reference problem is solved using the first heuristic allowing only lot shifts. The heuristics have been tested for different parameters.

In the second part, the problem of multi-item, multi-facility lot sizing problem is modified to include the unit production cost and under utilization and overtime cost of different facilities. An exact solution method based on Manne's [7] formulation has been used to test the efficiency and accuracy of the heuristic developed. A number of problems with varying parameters have been solved and analysis carried out.

CHAPTER I

INTRODUCTION

1.1 Material Requirements Planning:

Material Requirements Planning (MRP) is a computer based production planning and inventory control system. An MRP system is defined as a set of logically related procedures, decision rules and records designed to translate a master production schedule into time-phased net requirements and the planned coverage of such requirements for each component inventory item needed to implement this schedule. Master Production Schedule (MPS) outlines the production plan for all end items. MRP is also known as time-phased requirements planning. The key features of an MRP system are the time phasing of requirements generation of lower-level requirements, planned order releases and rescheduling capability. Time phasing of requirements simply establishes the time period in which work must be accomplished to meet the delivery date of the end item as stipulated in the MPS. Planned order releases indicate when orders should be placed for purchasing or manufacturing. When work cannot be accomplished on time, MRP can reschedule planned orders.

There is a basic difference in the application of traditional inventory control techniques and MRP. MRP systems are appropriate for dependent demand items. Demand for an item may be independent or dependent. Independent means no relationship

exists between the demand for an item and any other item, such as end products. Independent demand tends to be continuous and fluctuates because of random influences. In contrast, dependent means the demand for an item is directly related to or results from the demand for a higher level item. Dependent demand is not random but tends to occur in a lumpy manner at specific points in time. The lumpiness occurs because most manufacturing is in lots and all the items needed to produce the lots are usually withdrawn from inventory at the same time, not unit by unit. Thus although the demand for the final product may be continuous and independent, the demand for lower-level items composing the product tends to be discrete, derived and dependent.

Dependent demand items need not be forcasted, but can be calculated by the MRP system from MSP. In manufacturing organizations, most inventory items are dependent and should be controlled by an MRP system.

The effectiveness of MRP lies in the attainment of the following objectives concurrently.

1. Ensure the availability of materials, components and products for planned production and for customer delivery.
2. Maintain the lowest possible level of inventory.
3. Plan manufacturing activities, delivery schedules and purchase schedules.

1.2 Advantages and Limitations:

Since MRP systems demands handling of large volumes of data at high speed, its growth has parallelled developments in Computer Technology. Its origin, around 1960, was in line with the movement toward acceptance of quantitative management tools that used the data digestive power of computers.

Early spokesmen for MRP included G.W. Plossl, J.A.Orlicky and O.W. Wight. The first book on the subject written by J.A. Orlicky came out only in 1970. The MRP concepts were nurtured effectively by the American Production and Inventory Control Society (APICS) in the mid 1960 by what is called as MRP crusade. Their special report on Management of Lot Size Inventories is a milestone for the development of MRP Lot Sizing Techniques. Several organizations now offer software packages and consulting services for MRP implementation.

Riggs [1] while quoting Orlicky, suggests that successful MRP users enjoy a reduction in manufacturing inventory investment levels of 20% to 35% . Other claimed benefits are reduction in production and purchasing cost and improved delivery service. However, MRP implementations have not all been successful [2], and atleast one source indicates the proportion of successful MRP system is as low as 5 percent [3]. While carrying out a switchover from traditional inventory control system to an MRP system, human relations and technical difficulties have

to be fraught with. Lot of research work has been done to overcome these implementation difficulties.

Plossl and Wight [4] state that' ... there is tremendous application potential for MRP. There have been a number of highly successful companies and a great number of companies that have not really been able to use the tool effectively'. Further they point out that 'It is easy to see how companies can manage poorly without MRP, but it is hard to see how they can manage well without it.'

1.3 Focus and Organisation of the Thesis:

The quantitative aspect of MRP is mainly concerned with the problem of lot-sizing. Lot of work has already been done and reported in the literature on this subject. However, only limited attempts have been made to solve the problem of lot-sizing with capacity constraints. Most of these attempts are restricted to single facility environment. Karni [5,6] has considered the problem of single facility capacity constrained problem and has developed a heuristic for solving it. A survey of the literature indicates that, no meaningful solution methodology has been developed for lot-sizing problems with multi-facility capacity restrictions. In the present work we present two heuristics for such a problem.

In Chapter II, we present a formal statement of the problem of multi-item, multifacility requirements planning system and a mathematical formulation of the same.

In Chapter III, the heuristics developed for solving the problem stated in Chapter II are presented. Further the heuristics are compared for its performance and the results presented.

Finally in Chapter IV we have formulated a problem considering the overtime cost, under utilization cost and the production cost in addition to the setup cost and holding cost. Heuristic - I is modified and applied for solving this problem and it is tested by comparing with the exact solution method proposed by Manne [7].

CHAPTER II

PROBLEM FORMULATION

2.1 Capacity Consideration in Lot-Sizing:

Capacity refers to the production capacity of a work centre, department or facility. It is important to consider the capacity because of the following reasons,

1. Sufficient capacity is needed to provide the output for current and future customer demand,
2. Directly influences the cost of operations,
3. Represents a sizeable investment by the organisation.

A typical MRP system [8] is presented in Fig. 2.1. Before any detailed analysis of a given MPS takes place, a rough-cut capacity requirements plan is generated to determine whether the MPS is reasonable from a capacity point of view. Only aggregate measures of capacity requirements are used so that a variety of MPS can be quickly evaluated.

Once MPS that gives a satisfactory rough-cut capacity requirements plan is found, the bills of materials (BOM) is used to get the time phased requirement of all lower level items. Then the capacity required to satisfy this requirements schedule is calculated on the basis of standard hours of production for each work centre and time period. The idea is to check whether

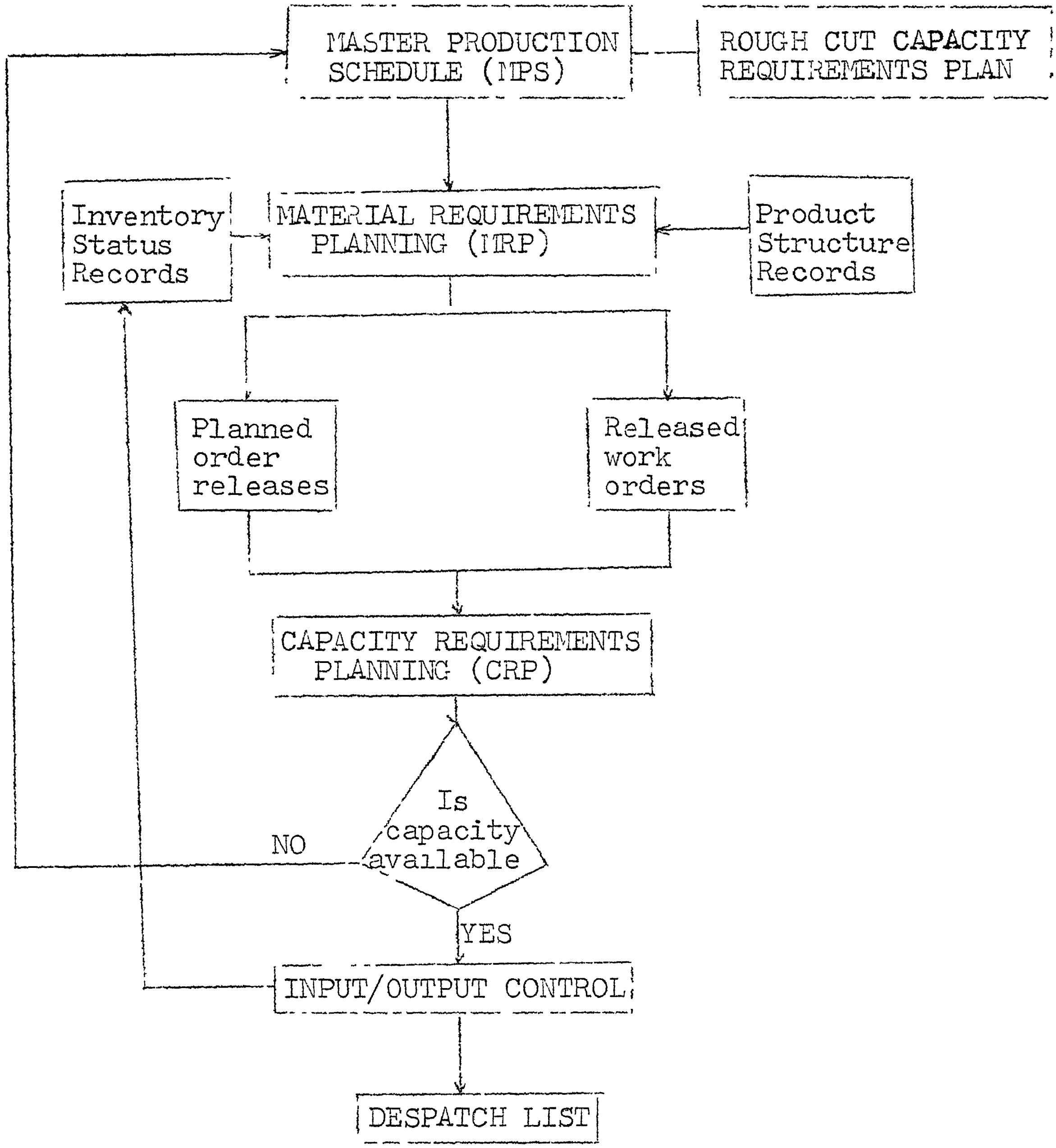

Fig. 2.1: An MRP System.

capacity is available in excess or is in shortage and then to decide the appropriate action to be taken. If capacity available is less than required capacity either the available capacity is augmented by incorporating overtime work, subcontracting, etc. or reduce capacity requirements by modifying the MPS. On the other hand if capacity available is greater than the required capacity then decisions in terms of product diversification or expansion are taken to utilize the excess capacity.

In the present context, we consider the case where facilities are restricted in terms of their capacities. In other words capacities of the different facilities are treated as uncontrollable or exogenous to the model. This type of capacity consideration is representative of the situation where expensive facilities are present and control of the capacity is not usually exercised and their capacity restrict the output. Further, it is assumed that capacity augmentation through overtime is not admissible.

2.2 Coordinated Replenishment

One of the basic assumption in most of the lot-sizing procedures is that, the different items under consideration are considered completely independent while calculating the lot-sizes. Silver [9] explained coordinated replenishment as a replenishment of a group or family of related items where in one or more members of the group are replenished at a time.

There are many ways by which this inter-relationship between members arise, for example, a common storage area, a common supplier, common facility for transportation/production etc.

There are many advantages and disadvantages in coordinated replenishment. The main advantages are:

1. Savings on ordering costs when the fixed cost of ordering/ production is shared by items in a group,
2. Savings on purchased/production items due to group discounts.
3. Savings on transportation costs,
4. Facilitates easier scheduling/procurement. For example co-ordinated handling of procurement is beneficial from the stand point of reduced work load etc.

The possible disadvantages of using co-ordinated replenishment procedure are,

1. When items are co-ordinated some will be ordered earlier than if they are considered independently.
2. An increase in system control costs like control review cost, computational cost etc.
3. Because we are not being able to work with items independently the flexibility of dealing with items in unusual situations will be reduced once the procedures for co-ordinated replenishment are established.

2.3 Problem Definition and Terminology:

The objective is to minimize the cost of the requirements schedule considering setup costs and holding costs. Setup costs comprise of major setup cost and minor setup cost. The problem of coordinated replenishment considered here involves a major setup cost (A) which will be incurred each time some items in a group are included in the replenishment schedule. In the production context we can say that this is the changeover cost associated with converting the facilities from the production of one group of products to another group. In the procurement context this is the cost of placing an order for a group of items which is not affected by the number of items in the group.

Next is the minor setup cost (S_k) which is incurred when item k is included in the replenishment. From the production point of view this minor setup cost is the cost of switching over production to item k from some other item in the group. From the procurement point of view this is the cost of adding item k in the requisition order.

Finally the holding cost (H_{tk}) is the cost of holding one unit of item k in time period t.

We assume that the capacity available (P_{ft}), which is the number of production hours on facility f available in time period t, is finite. We further assume that if the capacity required exceeds the available capacity, then we may shift the production of quantity to some other time periods.

The objective of our problem is to find the quantity of each item in the group to be replenished in each time period, so that the sum of setup costs and holding costs are minimized, while satisfying the capacity constraints.

2.4 Assumptions and Notations:

The following assumptions are made for the development of the model,

1. The demands of all the items in all the time periods in the planning horizon are known with certainty.
2. Back-logging is not allowed.
3. Lead time for production/procurement is negligible, however, if the items have a fixed lead time it can be taken care of by the model.
4. Inventories at the beginning and end of the planning horizon are zero, however, even if they are not zero, they can easily be taken care of by initializing these values. Beginning inventories can be reduced in the first period requirements and last period inventory can be added to the last period requirements.

The following notations are used.

T - Number of time periods in the planning horizon.

K - Number of items in the group.

F - Number of facilities that are used by these items under consideration.

A - Major set-up cost, Rs/setup.

S_k - Minor set-up cost for item k, Rs/setup of item k.

D_{tk} - Demand of item k in time period t, for $k = 1,\ldots, K$
$t = 1,\ldots, T$

X_{tk} - Order size of item k in time period t, for $k = 1,\ldots, K$
$t = 1,\ldots, T$

Y_{tk} - Ending Inventory of item k in time period t, for
$k = 1,\ldots, K$
$t = 1,\ldots, T$

H_{tk} - Cost of holding one unit of item k in inventory in time period t, Rs/item/period.

P_{ft} - Production capacity available in facility f in time period t, hours, for $f = 1,\ldots, F$; $t = 1,\ldots, T$

t_{fk} - Time, item k spends in facility f for one unit production, hours.

2.5 Problem Formulation:

The objective function is minimizing the costs of setup and the costs of carrying inventory.

Objective function,

$$\text{Minimize} \sum_{t=1}^{T} \left[A\,\delta\left(\sum_{k=1}^{K} X_{tk}\right) + \sum_{k=1}^{K} S_k \cdot \delta(X_{tk}) + \sum_{k=1}^{K} H_{tk}(Y_{tk})\right]$$

where, $\delta(\omega) = 0$ for $\omega = 0$

$\delta(\omega) = 1$ for $\omega > 0$

Various constraints are as given below,

1. Capacity constraints

$$\sum_{k=1}^{K} X_{tk} \cdot t_{fk} \leq P_{ft}, \quad \text{for } f = 1,\ldots, F; \; t = 1,\ldots, T$$

2. The material balance constraint,

$$\left\{\begin{array}{l}\text{The ending inventory}\\ \text{of previous time}\\ \text{period}\end{array}\right\} + \left\{\begin{array}{l}\text{Order size in the}\\ \text{present time period}\end{array}\right\}$$

$$= \left\{\begin{array}{l}\text{Ending Inventory in the}\\ \text{present time period}\end{array}\right\} + \left\{\begin{array}{l}\text{Demand in the}\\ \text{present time period}\end{array}\right\}$$

i.e. $$Y_{(t-1)k} + X_{tk} - Y_{tk} = D_{tk} \quad \text{for } t = 1,\ldots, T$$

$$k = 1,\ldots, K$$

3. Non-negative production quantity constraint,

$$X_{tk} \geq 0 \quad \text{for } t = 1,\ldots, T; \; k = 1,\ldots, K$$

4. Constraint to take care of back-logging,

$$Y_{tk} \geq 0 \quad \text{for } k = 1,\ldots, K; \; t = 1,\ldots, T$$

5. Constraint to ensure zero inventory at the beginning and end of the planning horizon,

$$\left.\begin{array}{l} Y_{ok} = 0 \\ Y_{tk} = 0 \end{array}\right\} \text{for } k = 1,\ldots, K$$

2.6 Complexity of the Problem:

An integer programming formulation of the problem is given below.

$$\text{Min. } TC = \sum_{t=1}^{T} [A \hat{X}_t + \sum_{k=1}^{K} S_{tk} Z_{tk} + \sum_{k=1}^{K} Y_{tk} H_{tk}]$$

subject to,

$$Y_{(t-1)k} + X_{tk} - Y_{tk} = D_{tk}, \quad k = 1, \ldots, K$$

$$X_{tk} - Z_{tk} M \leq 0, \quad k = 1, \ldots, K$$

$$\sum_{k=1}^{K} X_{tk} - \hat{X}_t M \leq 0$$

$$\sum_{k=1}^{K} t_{fk} X_{tk} \leq P_{ft}, \quad f = 1, \ldots, F$$

$$X_{tk} \geq 0 \quad , k = 1, \ldots, K$$

$$Y_{tk} \geq 0 \quad , k = 1, \ldots, K$$

$$\text{for } t = 1, \ldots, T$$

where,

M - A very large integer

Y_{tk} - Ending inventory of item k at time period t

$$\hat{X}_t = \begin{cases} 0 & \text{if } \sum_{k=1}^{K} X_{tk} = 0 \\ 1 & \text{otherwise} \end{cases} \qquad \text{for } t = 1, \ldots, T$$

$$Z_{tk} = \begin{cases} 0 & \text{if } X_{tk} = 0 \\ 1 & \text{otherwise} \end{cases} \qquad \begin{array}{l} \text{for } t = 1, \ldots, T \\ k = 1, \quad , K \end{array}$$

$$t_{fk} \geq 0$$

From the IP formulation we can see that there are (K+1)T, (0-1) integer variables and (K+F+1)T, slack variables. If we see the number of constraints there are KT, equality

constraints and (K+F+1)T, inequality constraints.

It is a well known fact that an IP problem grows complex as the number of constraints and variables increase. For our problem as K,T and F increase the number of constraints and variables increase rapidly. For a moderate size problem having $K = 6$, $T = 12$ and $F = 6$, there will be 84, (0-1) integer variables, 156 slack variables, 72 equality constraints and 156 inequality constraints. These problems are computationally complex in terms of CPU time, keeping this in view we have developed heuristics for solving this problem. The heuristic procedures are outlined in the next chapter.

CHAPTER III

SOLUTION METHODOLOGY

3.1 Heuristic Procedures:

A heuristic method as described by Gyorga Polya [10] is a method not regarded as final and strict, but as provisional and plausible only, whose purpose is to discover the solution of the present problem. Heuristic method does not claim that the solution is optimal. Heuristic methods are adopted mainly when the analytical methods are very complex in terms of modelling and solution. Heuristic method being simple quite often result in relatively lesser storage and CPU time requirements. In several situations, the solution from heuristic methods serve as initial solution to the analytical solution, thus reducing the complexity of the analytical solutions.

Considering the complexity of the problem, we have developed two heuristics for the present problem. In the first heuristic we assume that the requirement of an item in a particular time period can be shifted only as a whole lot. In the second heuristic we have assumed that we can split a lot and shift some quantity to the previous time period. Splitting the lot increases the possibility of getting a feasible solution.

3.2 Major Steps in the Heuristics:

The first step is to get a feasible solution. Since no back-logging is allowed items are scanned from the last time period back to time period two and minimum possible quantity required to eliminate infeasibility if any, is shifted to the previous time period. It may be noted that infeasibility here relates only to the capacity constraints. This shifts may result in an increase or reduction in total cost. Thus after the first pass, we may expect to get a feasible solution considering capacity limitations. If no feasible solution is found, the problem is a priori infeasible, a situation for which the production schedule perhaps has to be modified including the possibilities such as back logging, overtime, etc.

We assume that a feasible solution is obtained, then the next step is to improve the feasible solution by shifting item requirements to some previous period so as to achieve cost reduction.

3.3 Initial Feasibility Check:

Apart from the notations that have been defined in the previous chapter, the following notations are used,

C_{ft} - excess capacity available or needed in facility f in time period t.

$$C_{ft} = P_{ft} - \sum_{k=1}^{K} X_{tk}\, t_{fk} \quad \text{for} \quad \begin{array}{l} f = 1,\ldots, F \\ t = 1,\ldots, T \end{array}$$

positive C_{ft} means capacity is available while negative C_{ft} means capacity is needed.

E_{ft} - Additional capacity needed in facility f in time period t

$$E_{ft} = \begin{cases} 0 & \text{if } C_{ft} \geq 0 \\ ABS(C_{ft}) & \text{if } C_{ft} < 0 \end{cases}$$

Now that we have the requirements schedule D_{tk} with us. We can check if a solution can be obtained, by calculating the total capacity of each facility available over the planning horizon and the requirement of each facility over the planning horizon, for feasibility.

This leads to check if $\sum_{t=1}^{T} C_{ft} \geq 0$ for $f = 1, \ldots, F$. If the above conditions are satisfied, we can infer that a feasible solution may exist, otherwise the problem is infeasible.

3.4 Heuristic - I:

In this method shifts are made only as a whole lot of an item for getting a feasible solution. If a feasible solution is not obtained, we state the problem cannot be solved by this method.

3.4.1 Formulation:

First for getting a feasible solution shifts of items with maximum cost reduction and maximum capacity utilization are preferred. This is to ensure that by shifting minimum amount and saving maximum amount we are trying to get a feasible solution.

Relative capacity utilization of the shortage capacity by the items in a time period is given by, B_k

$$B_k = \sum_{f=1}^{F} (X_{tk} \cdot t_{fk}) \cdot \frac{E_{ft}}{\sum_{f=1}^{F} E_{ft}} \quad \text{for } k = 1, \ldots, K$$

Gain of each item in a time period, if it is shifted to the previous time period, is given by, $GAIN_k$

$$GAIN_k = S_k \cdot \sigma_k - X_{tk} \cdot H_{t-1,k} \quad \text{for } k = 1, \ldots, K$$

$$\text{where} \quad \sigma_k = \begin{cases} 0 & \text{if } X_{(t-1)k} \text{ or } X_{tk} = 0 \\ 1 & \text{if } X_{(t-1)k} \text{ and } X_{tk} > 0 \end{cases}$$

Now, representative factor,

$$BIK_k = B_k \cdot GAIN_k \quad \text{for } k = 1, \ldots, K$$

BIK is a representation of the capacity utilization of the excess facility capacities and gain in shifting the item. If the capacity in the time period under consideration is exceeded, we will shift one by one the item with maximum BIK value till the capacity available is enough for the production of the items left in the time period under consideration. This procedure is repeated for time period T to 2.

The above procedure may or may not give a feasible solution. If it does not give a feasible solution, we cannot solve the problem using this heuristic. If it gives a feasible solution we will apply two types of shifts to improve it. Shifts

are made only if they satisfy the capacity limitations and if there is some positive gain.

In the first type of shift, requirements of all items in a time period are shifted to some previous time period where some production is done. Assume that the requirements in time period j are shifted to time period i .

Setup cost saving is given by,

$$\text{SETCOST}_j = (A + \sum_{k=1}^{K} S_k . \sigma_k)$$

where $$\sigma_k = \begin{cases} 0, & \text{if } \overset{X}{\cancel{D}}_{jk} \text{ or } \overset{X}{\cancel{D}}_{ik} \text{ or both} = 0 \\ 1, & \text{if } \overset{X}{\cancel{D}}_{jk} \text{ and } \overset{X}{\cancel{D}}_{ik} > 0 \end{cases}$$

The cost calculated above can be easily explained as shown, since the requirements in both i and j are positive, one major setup cost A is saved, by shifting the production from period j to period i. D_{jk} and D_{ik} are the demands for item k in time period j and i. If both D_{ik} and D_{jk} are positive, we will save a minor setup cost S_k, otherwise there would not be any saving in minor setup cost.

Also due to the shift the items of time period j will be held in inventory from time period i to $(j-1)$. Hence the inventory carrying cost will also be incurred.

$$\text{CARRYCOST}_j = \sum_{k=1}^{K} \sum_{t=i}^{j-1} H_{tk} (X_{jk})$$

Now total gain will be,

$$TGAIN = SETCOST_j - CARRYCOST_j.$$

If TGAIN > 0, it is profitable to shift the entire requirement of time period j to time period i.

In the second type of shift, requirement of individual items in a time period are shifted to previous time periods, if there is a gain. If item k in time period j is shifted to time period i,

$$SGAIN_k = S_k - \sum_{t=i}^{j-1} H_{tk} (X_{jk}) ; \quad \text{for } k = 1, \ldots, K$$

If $SGAIN_k > 0$, then it is profitable to shift the individual item to a previous time period where some production occurs.

Now we can see the step by step procedure in the heuristic.

3.4.2 Steps in the Heuristic:

1. Start with the time phased requirements schedule obtained by lot-for-lot procedure (i.e.) each periods requirements is received in that time period.

2. Check feasibility by calculating C_{ft} matrix. If feasible proceed to the next step, otherwise the problem is infeasible.

3. Starting from last period to the time period t = 2 calculate BIK for each item in the time period and shift items with maximum BIK to time period t-1 for getting feasible solution.

4. Check if capacity requirement is met in the first time period,

$$\sum_{k=1}^{K} X_{1k}.t_{fk} \leq P_{f1}, \quad \text{for } f = 1, \ldots, F$$

If the above condition is satisfied go to the next step, otherwise we cannot get a solution using this method.

5. Start from time period $t = 2$.

6. Calculate, $TGAIN = SETCOST_t - CARRYCOST_t$.

If $TGAIN > 0$ go to the next step, otherwise go to (8).

7. Check capacity required to accommodate requirements of time period t in some previous time period where production is done.

If capacity is met add items of time period t to the previous time period where production in done, make $X_{tk} = 0$ for $k = 1, \ldots, K$, otherwise go to (8).

8. Calculate $SGAIN_k$ for $k = 1, \ldots, K$. Check the capacity and shift items in the order of maximum positive gain first till items with positive gain are no more possible to shift.

9. Repeat steps (6) to (8) till $t = T$.

3.5 Heuristic - II:

In this heuristic items are split-up and shifted for getting a feasible solution. Then for improving the feasible solution items are shifted as a whole to the left to gain setup cost and ending inventories are shifted to the right to gain carrying cost.

3.5.1 Formulation:

Initially, minimum quantity of each item is shifted to get a feasible solution. For a particular time period the amount to be shifted from different items is calculated by solving the simplex formulation given below,

$$\text{Minimize} \quad \sum_{k=1}^{K} \bar{\bar{x}}_{kt}$$

subject to,

$$\sum_{k=1}^{K} t_{fk} \cdot \bar{\bar{x}}_{kt} \geq E_{ft} \qquad \text{for } f = 1, \ldots, F \qquad (1)$$

$$\bar{\bar{x}}_{kt} \leq X_{tk} \qquad k = 1, \ldots, K \qquad (2)$$

$$\bar{\bar{x}}_{kt} \geq 0 \qquad (3)$$

where, $\bar{\bar{x}}_{kt}$ - quantity of item k in time period t to be shifted to the previous time period.

The objective function is minimizing the sum of the different items to be shifted. Constraint set (1) ensures that the facility utilized by the shifted quantity will be more than or equal to the shortage of facility capacity in that time period. Constraint set (2) puts an upper bound for the amount that can be shifted. Finally amount shifted should be non-negative.

For improving the feasible solution, two types of shifts are devised (1) shifting the ending inventory to future time period there by reducing holding cost. (2) Shifting item lot

as a whole leftward there by gaining setup cost.

The idea is first shift all the ending inventories possible from period i to j and shift back items as a lot from period j to i till capacity constraint in both period i and j are satisfied.

Ending inventory in period i that can be shifted to future period is given by,

$$Y_{ik} = \sum_{t=1}^{i} X_{tk} - \sum_{t=1}^{i} D_{tk} \quad \text{for } k = 1, \ldots, K.$$

Y_{ik} will be shifted only if there is some positive production in the next time period. Gain due to the shift of ending inventory is given by, $SGAIN_k$

$$SGAIN_k = \begin{cases} \sum_{t=i}^{j-1} Y_{tk} H_{tk} & \text{if } Y_{ik} < X_{ik} \\ S_k + \sum_{t=i}^{j-1} Y_{tk} H_{tk} & \text{if } Y_{ik} \geq X_{ik} \end{cases}$$

Now, suppose an item is shifted from period j to period i as a lot, the gain due to this left shift is given by, $GAIN_k$

$$GAIN_k = \begin{cases} S_k - \sum_{t=i}^{j-1} H_{tk} X_{jk} & \text{if } X_{ik} > 0 \\ - \sum_{t=i}^{j-1} H_{tk} X_{jk} & \text{if } X_{ik} = 0 \end{cases}$$

The shift devised is, possible ending inventories are added to the future time period where some production is done.

Then items from the later time period is shifted back to the time period under consideration in order of maximum gain till the capacity constraints are satisfied in both the time periods.

3.5.2 Steps in the Heuristic:

1. Start with the time phased requirements schedule obtained by lot-for-lot.

2. Calculate C_{ft} matrix and check initial feasibility. If feasible go to the next step, otherwise the problem is infeasible.

3. Starting from the last time period to t = 2, calculate $\bar{x}_{kt}$ for k = 1,..., K, using the simplex method proposed and add this quantities to the previous time period.

4. Check if $\sum_{k=1}^{K} X_{1k} . t_{fk} \leq P_{f1}$ for f = 1,..., F, is satisfied. If satisfied go to the next step, otherwise the problem is infeasible.

5. Starting from time period t = 1, try to perform shifts in time period t and t+1.

6. Calculate GAIN due to the shifts. If GAIN is positive shift all the items considered and try if items with positive gain can be further shifted, otherwise repeat step (5) and (6) for the next time period till t = T.

7. Calculate total cost of the schedule. Repeat steps (5) to (7) till the difference between two consecutive costs are zero.

3.6 Computational Experiment:

Our computational study is mainly done to find the better of the two heuristics. Hence we have limited the experimentation and a detailed study will be done with the model chosen, for applying in overtime consideration lot sizing.

The input data used for the study are presented in Table 1. We have tested the model for variations in major setup cost, minor setup cost and holding cost and the results presented in Table 2. It compares the percentage cost above the reference cost for variations of the above mentioned cost. The heuristic used for getting reference cost is presented in Appendix 1. This is obtained by relaxing the capacity constraints and then applying heuristic - I.

In Table 3, the cost of the production schedule and the CPU time taken by the heuristics are tested for different capacity level. This is mainly done to find how well the heuristic can arrive at a feasible solution with the limited capacity.

Finally the number of facilities, number of time periods and the number of products are varied and the CPU time of the heuristics are tested. This is presented in Table 4. Table 5 gives a numerical example for both the heuristics.

3.7 Results and Discussion:

It has been observed from Table 2 that in all the cases the cost of the production schedule obtained by heuristic - II

is higher than that for heuristic - I. Further when the setup costs are increased the cost above the reference cost also increase, where as when the holding cost is increased the deviation from reference cost decreases considerably. The above behaviour can be explained as follows. As the setup cost is increased the amount of shift made possible is increased, and as the holding cost is increased the economical shifts will be reduced. And since we are comparing the costs with the case where capacity is relaxed, this variation is highly noticeable. But if we look closely we find that heuristic-II performs badly compared to heuristic-I. This is mainly because, in heuristic II items are split-up and shifted which may reduce the possibility of shifting items in the previous time period. Also while applying feasibility procedure in heuristic-II, the costs are not considered.

The results obtained in Table 3 makes evident that even for very restricted capacities heuristic - II gives a feasible solution. Though in the cases where both the heuristic gave solution, the cost penalty is a bit high for heuristic-II, it assures a feasible solution in most of the cases.

Finally the CPU times are compared for variations in K, T and F and can be seen in Table 4. Here we find that there is not much increase in CPU time for heuristic - I, where as in heuristic - II the complexity of the simplex procedure increases

as K, T and F increases. Hence the CPU time for heuristic - II increase as K, T and F are increased.

3.8 Conclusion.

From the cost, CPU time and feasibility study done on both the heuristics, we find that it is economical both cost-wise and computation in terms of CPU time to use heuristic - I. But in case if the capacities are very much restricted that heuristic-I does not give a feasible solution we can use heuristic - II to get a feasible solution.

In the next chapter we have developed a model with over-time consideration and the performance of heuristic-I is compared with the exact solution.

Table 1

Major setup cost, A Rs/setup {15, 25, 35, 45, 55}

Minor setup cost, S_k Rs/setup {10, 15, 20, 25, 30}

Holding cost, H_{tk} Rs/Item/time period {0.4, 0.6, 0.8, 1.0, 1.2}

Number of facilities, F {2, 3, 4, 5, 6}

Number of products, K {2, 3, 4, 5, 6}

Number of time periods, T {6, 8, 10, 12}

Demand matrix, D_{tk}

38	45	20	65	78	26	40	39	42	0	12	36
12	50	55	45	50	41	10	2	17	39	23	48
39	14	49	36	15	10	31	3	53	18	50	30
42	0	12	36	17	39	23	48	71	8	62	43
17	39	23	48	53	18	50	30	37	86	11	9
53	18	50	30	42	0	12	36	91	7	82	63

Facility requirement time matrix, t_{fk}

1.3	1.4	1.6	1.3	1.9	2.1
1.0	0.8	1.7	2.4	1.2	1.7
2.0	2.0	1.0	0.9	0.4	1.1
1.4	0.8	1.9	0.3	2.4	1.3
1.7	1.6	2.1	1.7	1.1	1.0
0.8	2.2	0.6	1.3	1.2	0.6

Table 2: Average percentage cost above the reference cost for variations in major setup cost, minor setup cost and holding costs.

Cost	Variations for which tested	Average percent cost above reference cost	
		Heuristic I	Heuristic II
Major setup cost (A)	15	2.79	11.17
	25	4.35	15.89
	35	5.76	22.00
	45	7.87	29.17
	55	10.80	36.43
Minor setup cost (S_k)	10	3.29	12.80
	15	5.19	18.38
	20	8.83	24.44
	25	10.28	27.98
	30	13.54	32.51
Holding cost (H_{tk})	0.4	26.05	57.92
	0.6	9.50	30.23
	0.8	3.07	14.68
	1.0	1.22	8.52
	1.2	0.69	4.87

Table 3: Average cost of the production schedule and the CPU time taken by the two heuristics for different facility capacity level.

Capacity of production facility P_f, hours	Heuristic I		Heuristic II	
	CPU time milli sec.	Average cost, Rs.	CPU time milli sec.	Average cost, Rs.
200	--	--	78.8	833.46
250	--	--	93.3	806.79
300	69.08	736.82	98.08	781.76
350	60.04	724.89	88.24	775.64

Table 4: CPU time in milli secs. taken by both the heuristic for variation in the number of facility (F), number of product (K) and the number of time period (T).

Variation		CPU Time, milli seconds	
		Heuristic I	Heuristic II
Number of facility (F)	2	44.12	64.88
	3	69.08	98.08
	4	62.88	107.2
	5	65.36	142.52
	6	60.12	215.04
Number of products (K)	2	73.48	91.49
	3	44.12	64.38
	4	56.76	81.24
	5	69.08	104.4
	6	84.04	115.88
Number of time periods (T)	6	45.08	75.84
	8	69.08	98.08
	10	49.44	95.52
	12	52.32	106.76

Table 5: Numerical Example.

INPUT DATA:

$F = 3$, $T = 10$, $K = 3$, $A = 45$

$S_k = [15, 12, 15]$

$P_f = [300, 300, 300]$

$H_{tk} = [0.4, 0.6, 0.5]$

$$D_{tk} \quad \begin{matrix} 38 & 45 & 20 & 65 & 78 & 26 & 40 & 39 & 42 & 0 \\ 12 & 50 & 55 & 45 & 50 & 41 & 10 & 2 & 17 & 39 \\ 39 & 14 & 49 & 36 & 15 & 10 & 31 & 3 & 53 & 18 \end{matrix}$$

$$t_{fk} \quad \begin{matrix} 1.3 & 1.4 & 1.6 \\ 1.0 & 0.8 & 1.7 \\ 2.0 & 2.0 & 1.0 \end{matrix}$$

RESULTS:

Heuristic - I

$$X_{tk} \quad \begin{matrix} 38 & 65 & 0 & 65 & 78 & 66 & 0 & 81 & 0 & 0 \\ 12 & 50 & 55 & 45 & 50 & 51 & 0 & 19 & 0 & 39 \\ 53 & 0 & 49 & 51 & 0 & 41 & 0 & 56 & 0 & 18 \end{matrix}$$

Cost of production schedule = 749.5 rupees.

CPU time taken = 50 milli seconds.

Heuristic - II

$$X_{tk} \quad \begin{matrix} 38 & 65 & 0 & 65 & 78 & 26 & 40 & 39 & 42 & 0 \\ 12 & 50 & 55 & 45 & 50 & 51 & 0 & 19 & 0 & 39 \\ 53 & 0 & 49 & 51 & 0 & 10 & 34 & 0 & 71 & 0 \end{matrix}$$

Cost of production schedule = 805.2 rupees.

CPU time taken = 102 milli seconds.

CHAPTER IV

LOT SIZING WITH OVERTIME CONSIDERATION

In this chapter, we are going to develop the model where the regular time capacity is restricted. Capacity requirements beyond this level are met by overtime work. Further the production cost and the under utilization of facility costs are also considered.

4.1 Problem Definition and Terminology.

Our objective is to minimize the cost of the requirements schedule considering setup cost, holding cost, production cost, overtime cost and under utilization cost.

Setup costs and holding costs are the same as considered in Chapter II. Apart from that, production cost (CP_{tk}) is the cost of producing one unit of item k in time period t. Since a part of production cost is variable, we have considered the case where it varies over each time period.

Further the model considers a variable overtime cost (CO_{ft}), which is the cost of using unit overtime in hours in facility f for time period t and a variable under utilization cost (CO_{ft}), which is the cost incurred because of the under utilization of a unit of facility f in time period t. Under utilization cost

is incurred because of the investment in equipment, deterioration cost of equipment,etc.

The objective of our problem is to find the requirements schedule so that the total cost is minimized.

The following notations are used apart from that defined in Chapter II.

R_{ft} - Regular time capacity available in facility f in period t, hours.

O_{ft} - Overtime utilization required in facility f in period t, hours.

U_{ft} - Under utilization of facility f in period t, hours.

CP_{tk} - Cost of production of a unit of item k in period t,Rupees.

CU_{ft} - Cost incurred due to the under utilization of per unit of facility f in period t, Rupees.

CO_{ft} - Cost of utilizing per unit of overtime in facility f in time period t, Rupees.

Overtime utilization is the amount of facility f in hours required in period t that cannot be met by the regular time capacity available in that time period. Some additional costs are incurred due to overtime utilization.

The assumptions stated in Chapter II holds good here also.

4.2 Problem Formulation:

The objective function is minimizing the costs of setup, holding, production, overtime and under utilization.

Objective function,

$$\text{Minimize}, \sum_{t=1}^{T} \Big[A\,\delta\Big(\sum_{k=1}^{K} X_{tk}\Big) + \sum_{k=1}^{K} S_k\,\delta(X_{tk}) + \sum_{k=1}^{K} H_{tk}\,Y_{tk}$$

$$+ \sum_{k=1}^{K} CP_{tk}\,X_{tk}\Big] + \sum_{f=1}^{F} \sum_{t=1}^{T} \big[CU_{ft}\,U_{ft} + CO_{ft}\,O_{ft}\big]$$

where, $$\delta(\omega) = \begin{cases} 0 & \text{if } \omega = 0 \\ 1 & \text{if } \omega > 0 \end{cases}$$

Various constraints are as given below,

1. Capacity balance equations,

$$\left.\begin{aligned} \sum_{k=1}^{K} X_{tk}\,t_{fk} + U_{ft} - O_{ft} &= R_{ft} \\ U_{ft}\,O_{ft} &= 0 \end{aligned}\right\} \quad \begin{aligned} &\text{for } f = 1,\ldots, F \\ &\text{for } t = 1,\ldots, T \end{aligned}$$

2. Material balance constraint,

$$Y_{(t-1)k} + X_{tk} - Y_{tk} = D_{tk} \qquad \text{for } t = 1,\ldots, T, \quad k = 1,\ldots, K$$

3. Positive production qualtity,

$$X_{tk} \geq 0 \qquad \text{for } t = 1,\ldots, T, \quad k = 1,\ldots, K.$$

4. Constraint to take care of backlogging,

$$Y_{tk} \geq 0 \qquad \text{for } t = 1,\ldots, T, \quad k = 1,\ldots, K$$

5. Ensure zero inventory at the beginning and end of the planning horizon,

$$Y_{0k} = 0 \qquad \text{for } k - 1, \ldots, K.$$
$$Y_{Tk} = 0$$

4.3 Exact Solution Methodology.

The exact solution method is used to solve the problem mainly for comparing the performance of the heuristic cost-wise and computationally (CPU time).

We are using Manne's formulation [7]. This model considers all the dominant production sequence and selects the one with the least cost. It assumes that an item lot cannot be split. An item having net requirements for example 20, 15 and 40 in 3 periods has four (2^{3-1}) dominant sequence. They are (20, 15, 40), (20, 55, 0), (35, 0, 40) and (75, 0, 0). So the number of combinations to be checked for a schedule having T periods and K products are $(2^{T-1})^K$. For a problem having 10 time periods and 2 products, the number of combinations to be checked is 262144.

4.4 Formulation of the Heuristic.

Two types of shifts are devised here, the first one shifts whole items of a time period to some previous time period where some items are manufactured. The cost changes due to this shift of items from period j to period i are shown below.

Sl. No.	Increase in cost	Sl. No.	Decrease in cost
1.	Carrying cost of items in time period j from period i to period (j-1).	5.	Setup cost gained due to the shift.
2.	Overtime in period i in case the shift results in overtime in period i.	6.	Overtime in j relieved in case there is overtime in j prior to the shift.
3.	Under utilization cost in period j caused due to the shift.	7.	Regular time capacity utilised in period i if some regular time capacity is available in period i.
4.	Production cost if the items of period j are manufactured in period i.	8.	Production cost in j relieved due to the shift.

Cost changes in time period i are shown below,

Excess capacity available in each facility in time period i

$$E_{fi} = \begin{cases} 0, & \text{if } R_{fi} - \sum_{k=1}^{K} X_{ik} - t_{fk} \leq 0 \quad \text{for } f = 1, \ldots, F \\ R_{fi} - \sum_{k=1}^{K} X_{ik} . t_{fk}, & \text{otherwise.} \end{cases}$$

Capacity required by items in time period j,

$$P_{fj} = \sum_{k=1}^{K} X_{jk} . t_{fk} \qquad \text{for } f = 1, \ldots, F.$$

Now find DIFF = $E_{fi} - P_{fi}$.

If the DIFF is positive or zero then we will gain some cost due to the utilization of regular time in i.

$$(7) = P_{fj} \times CU_{fi}$$

If the DIFF is negative then we may gain some cost by utilizing regular time in i and incur some cost by requiring overtime capacity.

$$(7) = E_{fi} \times CU_{fi}$$

$$(2) = ABS\,(E_{fi} - P_{fj}) \times CO_{fi}$$

Production cost added in period i due to the shift.

$$(4) = \sum_{k=1}^{K} X_{jk} \cdot CP_{ik}$$

Cost changes in time period j are shown below.

Now the production facility utilized by items in time period j is reduced from the regular time capacity available.

If the regular time capacity is still positive or zero, then some cost is increased due to the relieving of regular time capacity in period j .

$$(3) = P_{fj} \times CU_{fj}$$

If the capacity required is more than the regular facility capacity in period j then some cost will increase because regular time capacity is relieved in period j and some cost will reduce because overtime is relieved in j .

$$(3) = R_{fj} \times CU_{fj}$$

$$(6) = ABS\,(R_{fi} - P_{fi}) \times CO_{fi}$$

Finally due to the shift some carrying cost will be incurred and some set-up cost may be reduced.

Carrying cost, (1) $= \sum_{k=1}^{K} \sum_{t=1}^{j-1} X_{jk} \times H_{tk}$

Setup cost, (5) $= A + \sum_{k=1}^{K} S_k \sigma_k$

where,

$$\sigma_k = \begin{cases} 0 & \text{if } X_{i,k} \text{ or } X_{j,k} \text{ or both } = 0 \\ 1 & \text{if } X_{i,k} \text{ and } X_{j,k} > 0 \end{cases}$$

Total cost change due to the shift is given by,

TGAIN = INCREASE IN COST - DECREASE IN COST.

(i.e.) TGAIN = ((5) + (6) + (7) + (8)) - ((1) + (2) + (3) + (4)).

If the gain is positive then, it will be economical to manufacture items of time period j in time period i.

In the second type of shift, individual item of a time period are shifted to previous time periods if there is a cost improvement in the production schedule. The cost changes due to the shift of item m from period j to i are given below.

Sl. No.	Increase in Cost	Sl. No.	Decrease in Cost
1.	Carrying cost of item m from period i to period (j-1).	5.	Setup cost saved.
2.	Overtime utilized in period i in case the shift results in overtime in i.	6.	Regular time utilized in i if it is available in period i.
3.	Regular time relieved in j, due to the shift.	7.	Overtime relieved in j in case there is over time in j prior to the shift.
4.	Production cost of item m in period i.	8.	Production cost of item m in period j.

The different costs involved are calculated as follows,

$$\text{Carrying cost} \quad (1) \;=\; \sum_{t-i}^{j-1} X_{jm} \times H_{tm}$$

$$\text{Setup cost} \quad (5) \;=\; S_m \cdot \sigma_m$$

where,

$$\sigma_m = \begin{cases} 0 & \text{if } X_{i,m} \text{ or } X_{j,m} \text{ or both} = 0. \\ 1 & \text{if } X_{i,m} \text{ and } X_{j,m} > 0 \end{cases}$$

Cost changes in period i are shown below,

Calculate,

$$E_{fi} = \begin{cases} 0 & \text{if } (R_{fi} - \sum_{k=1}^{K} X_{ik} \cdot t_{fk}) < 0 \\ (R_{fi} - \sum_{k=1}^{K} X_{ik} \cdot t_{fk}), & \text{otherwise} \end{cases} \qquad \text{for } f = 1, \ldots, F$$

$$P_f = X_{jm} \cdot t_{fm} \qquad \text{for } f = 1, \ldots F$$

Sl. No.	Increase in Cost	Sl. No.	Decrease in Cost
1.	Carrying cost of item m from period i to period (j-1).	5.	Setup cost saved.
2.	Overtime utilized in period i in case the shift results in overtime in i.	6.	Regular time utilized in i if it is available in period i.
3.	Regular time relieved in j, due to the shift.	7.	Overtime relieved in j in case there is over time in j prior to the shift.
4.	Production cost of item m in period i.	8.	Production cost of item m in period j.

The different costs involved are calculated as follows,

Carrying cost (1) $= \sum_{t-i}^{j-1} X_{jm} \times H_{tm}$

Setup cost (5) $= S_m \cdot \sigma_m$

where,

$$\sigma_m = \begin{cases} 0 & \text{if } X_{i,m} \text{ or } X_{j,m} \text{ or both} = 0. \\ 1 & \text{if } X_{i,m} \text{ and } X_{j,m} > 0 \end{cases}$$

Cost changes in period i are shown below,

Calculate,

$$E_{fi} = \begin{cases} 0 & \text{if } (R_{fi} - \sum_{k=1}^{K} X_{ik} \cdot t_{fk}) < 0 \\ (R_{fi} - \sum_{k=1}^{K} X_{ik} \cdot t_{fk}), & \text{otherwise} \end{cases} \quad \text{for } f = 1, \ldots, F$$

$$P_f = X_{jm} \cdot t_{fm} \qquad \text{for } f = 1, \ldots F$$

Find $(E_{fi} - P_f)$, this gives the difference of excess facility capacity in period i and the capacity required for the shift.

Now,

If $(E_{fi} - P_f) \geq 0$, then cost incurred is

$$(6) = P_f \times CU_{fi}$$

Otherwise,

$$(6) = E_{fi} \times CU_{fi}$$

$$(2) = (P_f - E_{fi})\ CO_{fi}$$

Production cost will also be incurred in i,

$$(4) = X_{jm} \times CP_{im}.$$

Cost changes in period j are given below.

Due to the shift of item m from time period j to i some overtime capacity may be relieved in period j, in that case cost gain,

$$(7) = P_f \times CO_{fj}$$

In some case some overtime and regular time capacities are relieved, in that case there will be a reduction in some cost and an increase in some other cost,

(i.e.) increase in cost, $(3) = C_{fi} \times CU_{fj}$

reduction in cost, $(7) = (P_f - C_{fi})\ CO_{fi}$

If because of the shift only regular time capacity is relieved then increase in cost is,

$$(3) \quad = \quad P_f \times CU_{fj}$$

Also because of the shift production cost in period j is reduced, by,

$$(8) \quad = \quad X_{jm} \times CP_{jm}$$

Now, GAIN $= ((1) + (2) + (3) + (4)) - ((5) + (6) + (7) + (8))$.

If the gain is > 0 it is economical to shift item m from time period j to period i.

4.5 Steps in the Heuristic:

1. Start with the lot-for-lot schedule.
2. Calculate TGAIN for time period j and if TGAIN > 0 shift it to period i where some production is done. Repeat step (2) for $j = 2$ to T.
3. Calculate GAIN for individual shift and shift item with maximum gain in the time period under consideration. Repeat Step (3), till no more gain in time period j is positive.
4. Repeat step (3) for $j = 2$ to T.
5. Calculate cost of the schedule.

4.6 Computational Experiment:

As has been seen the complexity of the exact solution method increases as the number of time periods and the number of items increases. Hence keeping $K = 2$, the number of time periods has been varied for $T = 4, 6, 8, 10$ and the average time taken by the heuristic and exact method, and cost data are presented in Table 8. Similarly keeping the number of time periods, $T = 4$ the number of products are varied for $K = 2, 3, 4, 5$ and the results presented in Table 9

In Table 10 the performance of the heuristic has been tested for different cost patterns. (viz., increasing, decreasing and random). This is done by varying, holding cost, cost of overtime, cost of under utilization and the production cost.

Finally the model has been tested for different values of holding cost, setup cost, production cost, under utilization cost and overtime cost, varying one at a time and the results presented in Table 11. In total we have tested 408 problems. The different input data used are presented in Table 6. One example problem has been presented in Table 7.

4.7 Results and Discussion.

4.7.1 Solution Accuracy

From Table 8, we find that, the overall performance of the heuristic in terms of solution accuracy is comparably good. For $T - 6$ and $T = 10$, the costs are off set by some values which

deviate very much from the optimal. From Table 9, we find that for low values of K, the heuristic performs well, however for K = 4 and K = 5, the deviation from optimal cost is significantly high.

From Table 10, representing the performance for different cost patterns we find that the deviation from exact solution is not much and the performance of the heuristic is good. The average cost above the optimal cost is a bit higher for the case of random cost pattern in overtime. This is due to the offset of average by one value, which deviates very much from optimal.

As can be seen from Table 11, the deviation from exact solution is not varying much as the major setup cost and the cost of overtime are varied. But when the holding cost, cost of production and the under utilization costs are increased the deviation decreases considerably where as when the minor setup cost is increased the deviation from exact solution also increases. From this we can conclude that whenever the shifts are restricted the heuristic performs well.

4.7.2 CPU Times:

From Table 8, we find that as the number of time periods increase the CPU time for exact solution increases considerably, whereas the CPU time for the heuristic increased by only very little amount. The same fact can be observed for increase in K also. This is due to the fact that the complexity of the

exact solution is of the order of $(2^{T-1})^K$ where as for the heuristic it is of the order of KT.

Finally the reasons, why the heuristic gives suboptimal solutions are investigated and the main reasons are summarised.

1. In the heuristic first we are trying to shift items of a time period as a whole. Because of this shift, the gain due to the shift of individual items may be flatened out (i.e.) it may be economical to shift some items rather than shifting items of a time period as a whole.

2. Because in both the shifts we are moving from $t = 2$ to $t = T$ we might have shifted some quantity which prevents the more economical shift in a later time period.

4.8 Conclusion:

The heuristic in general performs well both in terms of solution accuracy and computational time. It is very easy to understand and apply.

We have considered the main costs in a production environment. Hence the model has practical relevance also. The model can take care of overtime capacity limitation too . This is done by assuming very high overtime cost beyond the allowed overtime.

Table 6: Input Data Used.

$K = \{2,3,4,5,6\}$, $T = \{4,6,8,10,12\}$

$F = \{2,3,4,5,6\}$, $A = \{25,35,45,55\}$

Minor setup cost - S_k $\{15,30,12,13,25,9\}$

Facility capacity - P_{ft}

180	120	256	118	95	160	130	130	210	122	146	200
90	85	200	150	145	150	102	110	85	190	70	95
120	200	95	175	120	180	120	180	70	145	165	150
112	76	120	140	86	190	188	160	79	100	120	115
80	120	89	149	156	170	180	113	98	79	200	198
200	156	178	89	96	140	50	78	120	115	111	135

Time in facility t_{fk}

1.3	1.4	1.6	1.3	1.9	2.1
1.0	0.8	1.7	2.4	1.2	1.7
2.0	2.0	1.0	0.9	0.4	1.1
1.4	0.8	1.9	0.3	2.4	1.3
1.7	1.6	2.1	1.7	1.1	1.0
0.8	2.2	0.6	1.3	1.2	0.6

Requirements matrix -- D_{tk}

38	45	20	65	78	26	40	39	42	0	12	36
12	50	55	45	50	41	10	2	17	39	23	98
39	14	49	36	15	10	31	3	53	18	50	30
42	0	12	36	17	39	23	48	71	8	62	43
17	39	23	48	53	18	50	30	37	86	11	9
53	18	50	30	42	0	12	36	91	7	82	63

Holding cost - H_{tk}

0.76	0.34	0.25	0.36	0.11	0.56	0.54	0.40	0.45	0.90	0.68	0.13
0.32	0.21	0.35	0.16	0.88	0.22	0.11	0.39	0.14	0.79	0.11	0.13
0.52	0.31	0.45	0.56	0.27	0.28	0.98	0.36	0.56	0.36	0.15	0.92
0.11	0.76	0.56	0.34	0.92	0.36	0.56	0.54	0.38	0.28	0.29	0.52
0.26	0.48	0.22	0.28	0.12	0.49	0.48	0.68	0.29	0.56	0.76	0.73
0.41	0.37	0.08	0.31	0.76	0.56	0.22	0.11	0.18	0.79	0.66	0.09

Table 1 contd...

Cost of production - CP_{tk}

1.25	1.89	1.65	2.20	0.60	0.90	1.76	1.84	1.22	1.40	1.20	1.90
1.89	1.50	1.29	1.49	2.10	1.40	1.70	0.97	0.65	0.40	1.80	1.40
0.95	1.50	1.34	0.85	0.46	0.98	1.20	1.15	2.00	1.18	1.70	1.60
0.86	0.98	1.20	2.10	1.87	1.49	1.67	1.73	1.20	1.11	0.98	1.02
0.25	0.40	1.20	1.35	1.28	0.78	0.64	0.98	1.20	1.17	0.98	2.01
1.67	1.56	2.01	1.35	1.36	2.04	2.07	1.98	0.46	0.78	0.98	1.21

Cost of under utilization - CU_{ft}

0.32	0.10	0.22	0.36	0.34	0.07	0.12	0.78	0.61	0.22	0.46	0.37
0.03	0.20	0.09	0.21	0.54	0.18	0.37	0.22	0.78	0.16	0.09	0.40
0.71	0.89	0.14	0.26	0.17	0.78	0.32	0.12	0.72	0.65	0.11	0.43
0.09	0.76	0.52	0.31	0.09	0.18	0.28	0.72	0.36	0.76	0.12	0.62
0.20	0.11	0.43	0.72	0.46	0.65	0.31	0.65	0.11	0.38	0.26	0.71
0.28	0.26	0.16	0.18	0.28	0.47	0.46	0.54	0.25	0.29	0.37	0.22

Cost of overtime - CO_{ft}

1.24	1.36	2.78	3.08	1.76	1.94	2.20	3.80	0.78	2.45	0.98	1.12
2.76	1.25	2.00	1.00	0.75	1.53	2.68	2.98	1.89	1.39	0.68	0.43
0.97	2.34	2.94	1.46	1.39	0.50	1.29	2.63	1.11	2.22	2.89	1.44
0.63	1.36	2.30	1.63	2.12	0.12	1.39	2.63	1.38	2.69	0.69	1.43
1.38	2.54	0.54	1.13	1.27	3.00	2.89	2.79	1.62	0.78	0.53	0.62
2.40	1.44	2.14	2.01	1.00	0.79	0.65	1.25	2.00	2.25	1.74	3.00

Increasing holding cost - H_{tk}

0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32
0.09	0.11	0.13	0.15	0.17	0.19	0.21	0.23	0.25	0.27	0.29
0.10	0.13	0.16	0.19	0.22	0.25	0.28	0.31	0.34	0.37	0.40
0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52
0.01	0.06	0.11	0.16	0.21	0.26	0.31	0.36	0.41	0.46	0.51
0.02	0.07	0.12	0.17	0.22	0.27	0.32	0.37	0.42	0.47	0.52

Increasing cost of production - CP_{tk}

1.02	1.25	1.36	1.45	1.56	1.78	1.92	2.05	2.42	2.78	3.02	3.25
0.78	0.98	1.21	1.36	1.49	1.65	1.68	1.78	1.82	1.89	1.95	2.03
1.25	1.31	1.42	1.65	1.78	1.82	1.83	1.95	2.07	2.12	2.20	2.28
1.31	1.42	1.48	1.52	1.59	1.68	1.76	1.82	1.89	1.97	2.27	2.68
1.06	1.12	1.18	1.22	1.26	1.32	1.36	1.42	1.46	1.52	1.58	1.66
0.76	0.87	0.92	1.08	1.16	1.21	1.39	1.48	1.59	1.69	1.72	1.93

Table 1 contd...

Increasing cost of under utilization - CU_{ft}

0.11	0.13	0.19	0.22	0.26	0.32	0.34	0.39	0.42	0.46	0.52	0.56
0.07	0.07	0.12	0.14	0.19	0.22	0.29	0.32	0.33	0.38	0.44	0.47
0.21	0.24	0.26	0.32	0.39	0.41	0.46	0.52	0.58	0.59	0.66	0.72
0.19	0.22	0.26	0.29	0.36	0.39	0.47	0.56	0.68	0.77	0.79	0.87
0.22	0.23	0.34	0.42	0.46	0.48	0.57	0.58	0.60	0.65	0.71	0.76
0.16	0.19	0.22	0.26	0.29	0.34	0.38	0.43	0.49	0.54	0.56	0.59

Increasing cost of overtime - CO_{ft}

1.26	1.35	1.46	1.54	1.59	1.62	1.76	1.79	1.85	1.89	1.92	2.01
0.98	1.02	1.08	1.12	1.16	1.22	1.28	1.32	1.36	1.39	1.42	1.46
2.10	2.12	2.20	2.28	2.32	2.36	2.44	2.52	2.56	2.60	2.66	2.68
1.12	1.20	1.26	1.32	1.37	1.43	1.49	1.55	1.59	1.64	1.67	1.77
0.52	0.67	0.75	0.83	0.89	0.97	1.07	1.13	1.21	1.27	1.31	1.33
0.03	0.17	0.26	0.35	0.46	0.59	0.66	0.77	0.83	0.92	1.07	1.12

Increasing requirements matrix - D_{tk}

25	30	35	40	45	50	55	60	65	70	75	80
10	17	24	31	38	45	52	59	66	73	80	87
2	10	18	26	34	42	50	58	66	74	82	90
7	11	15	19	23	27	31	35	39	43	47	51
15	20	25	30	35	40	45	50	55	60	65	70
20	25	30	35	40	45	50	55	60	65	70	75

Table 7: Numerical Example.

INPUT DATA:

$K = 2,\ T = 8,\ F = 3,\ A = 45$

$$D_{tk} \quad \begin{matrix} 38 & 45 & 20 & 65 & 78 & 26 & 40 & 39 \\ 12 & 50 & 55 & 45 & 50 & 41 & 10 & 2 \end{matrix}$$

For all other variables, random values from Table 1 are used.

OUTPUT :

Exact Solution.

$$X_{tk} \quad \begin{matrix} 83 & 0 & 85 & 0 & 78 & 66 & 0 & 39 \\ 12 & 105 & 0 & 95 & 0 & 53 & 0 & 0 \end{matrix}$$

Cost of production schedule = 2237.948 rupees

CPU time taken = 35858 milli seconds.

Heuristic Solution:

$$\hat{X}_{tk} \quad \begin{matrix} 38 & 65 & 0 & 65 & 78 & 66 & 0 & 39 \\ 62 & 0 & 150 & 0 & 0 & 53 & 0 & 0 \end{matrix}$$

Cost of production schedule = 2322.458 rupees

CPU time taken = 33 milli seconds.

Table 8: Average CPU time and Percentage Cost above Optimal for Variation in T.

No. of time periods	Average CPU time by heuristic method, milli secs.	Average CPU time by exact solution, milli secs.	Mean % cost above optimal	Max % cost above optimal
T = 4	14	70	14.96	39.07
T = 6	19	1698	22.70	56.65
T = 8	23	35857	9.93	28.53
T = 10	42	712503	19.03	35.73

Table 9: Average CPU time and Percentage Cost above Optimal for Variation in K.

No. of products	Average CPU time by heuristic method, milli secs.	Average CPU time by exact solution, milli secs.	Mean % cost above optimal	Max % cost above optimal
K = 2	14	70	4.96	39.07
K = 3	15	783	3.77	20.68
K = 4	31	7966	15.99	53.84
K = 5	45	76915	14.72	66.72

Table 10: Average CPU time and percentage cost above optimal for different cost patterns.

Cost	Pattern	Avg. CPU time by heuristic methods, milli secs.	Avg. CPU time by exact solution, milli secs.	Mean percent cost above optimal	Max. percent cost above optimal
Holding cost (H_{tk})	increase	25	74740	3.59	4.36
	decrease	22	74686	2.81	3.26
	random	22	74625	2.65	3.47
Cost of production (CP_{tk})	increase	20	74652	2.35	3.08
	decrease	21	74640	1.27	1.69
	random	23	74634	3.42	8.62
Cost of under utilization (CU_{ft})	increase	22	74647	1.60	2.14
	decrease	23	74655	2.20	2.53
	random	26	74638	2.71	9.21
Cost of overtime (CO_{ft})	increase	23	74648	2.58	3.75
	decrease	26	74730	2.00	2.92
	random	25	74744	5.69	28.89

Table 11: Percentage deviation from exact solution for different values of the costs considered.

Cost	Values for which tested	Percent deviation from exact solution
Major setup cost (A)	25	2.38
	35	2.32
	45	2.27
	55	2.22
Minor setup cost (S_k)	15	1.61
	20	2.09
	25	2.58
	30	3.04
Holding cost (H_{tk})	0.25	3.07
	0.50	2.11
	0.75	1.18
	1.00	0.28
Cost of production (CP_{tk})	0.5	2.57
	1.0	2.21
	1.5	1.94
	2.0	1.73
Cost of under utilization (CU_{ft})	0.25	2.01
	0.50	1.79
	0.75	1.59
	1.00	1.39
Cost of overtime (CO_{ft})	0.75	2.76
	1.50	2.77
	2.25	2.78
	3.00	2.36

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APPENDIX I

In the heuristic proposed below no capacity constraints are considered. The objective is reducing the sum of setup costs and carrying costs.

Two types of shifts are devised. First one is shifting items of a time period as a whole, if there is gain. The second one is shifting individual items to some previous time period, if there is gain. Cost change due to the first shift is given below,

$$SETC_j = (A + \sum_{k=1}^{K} S_t . \epsilon_k)$$

where,

$$\epsilon_k = \begin{cases} 0 & \text{if } D_{ki} \text{ or } D_{kj} \text{ or both } = 0 \\ 1 & \text{if } D_{ki} \text{ and } D_{kj} > 0 \end{cases}$$

$SETC_j$ - Setup cost gained due to the shift of items from period j to i.

$$HOLD_j = \sum_{k=1}^{K} \sum_{t=i}^{j-1} D_{jk} . H_{tk}$$

$HOLD_j$ - Cost of holding items of period j in inventory from period i to j-1.

Now, $GAIN_j = SETC_j - HOLD_j$

If $GAIN_j > 0$, then items of period j are shifted to period i.

Cost changes in shifting individual items,

$$GAIN_{kj} = S_k - \sum_{t=i}^{j-1} X_{jk}.H_{tk}, \quad \text{for } k = 1, \ldots, K$$

where,

D_{tk} - Demand of item k in period t.

X_{tk} - Requirements of item k in period t.

Steps in the Heuristic:

1. Initially the coverage of the demands for all items are made by using the lot-for-lot approach.

2. Start with time period t = 2.

3. Calculate $GAIN_t$. If $GAIN_t > 0$, Go to Step (4), otherwise t = t+1, Go to Step (3).

4. Add X_{tk} to the previous time period where some positive production is done, make $X_{tk} = 0$, for k = 1,..., K.

5. Repeat steps (3) and (4) till all time periods are covered.

6. Start with time period t = 2.

7. Calculate $GAIN_{kt}$ and shift if it is positive. Repeat this upto t = T.

8. Calculate the cost of the production schedule.

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